

Heating the coffee by looking at it. Or why quantum measurements are physical processes

Pablo Echenique-Robba^{*,1,2,3,4,5}

¹*Instituto de Química Física Rocasolano, CSIC, Madrid, Spain*

²*Instituto de Biocomputación y Física de Sistemas Complejos (BIFI), Universidad de Zaragoza, Spain*

³*Zaragoza Scientific Center for Advanced Modeling (ZCAM), Universidad de Zaragoza, Spain*

⁴*Departamento de Física Teórica, Universidad de Zaragoza, Spain*

⁵*Unidad Asociada IQFR-BIFI, Madrid-Zaragoza, Spain*

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Abstract

Using a very simple *Gedankenexperiment*, I remind the reader that (contrary to what happens in classical mechanics) the energy of a quantum system is *inevitably* increased just by performing (some) textbook measurements on it. As a direct conclusion, this means that some measurements require the expenditure of a finite amount of energy to be carried out. I also argue that this makes it very difficult to regard measurements as disembodied, immaterial, informational operations, and it forces us to look at them as physical processes just like any other one.

Keywords: measurements, quantum mechanics, energy, quantum information

1 Introduction

In everyday life, we know that a cold coffee mug does not heat up simply by looking at it. We have to interact in some stronger way with it than just looking if we want to drink the coffee hot; for example, we have to put it in the microwave. Similarly, we know that no other type of energy can be transferred to a physical system *just by finding out something about it*: watching at the blinking dot in a radar screen does not change the speed of the corresponding plane, looking at Mars through the telescope does not alter the weather there, and watching the football fly in the TV does not increase the probability that it ends up scoring a goal—no matter how much the fans would like it (or even assume it).

Classical mechanics underwrites this intuition—this “folk physics”—formally and mathematically. In classical mechanics it is tacitly assumed that the possible disturbance on the system that may be produced by measuring any of its physical properties can be in principle reduced to naught. That is, classical mechanics does admit the possibility that measurements cause a non-negligible disturbance (for example, if you measure the position of a football by

* pablo.echenique.robba@gmail.com — <http://www.pabloecheniquerobba.com>

throwing tennis balls at it and looking which ones bounce back at you), but nothing in the formalism precludes you from designing better and better measuring devices until you make the disturbance vanishingly small. Famously, this is *not* the case in quantum mechanics; as I will try to show here in a simple but dramatical example.

What happens *exactly* when we perform a measurement on a quantum system is still an open question, but one thing is clear: it is not at all like the classical case. The literature of quantum foundations is a bit messy at the moment with many different accounts competing for prime time in the journals (Eichenique-Robba, 2013), and most of them differ in the characterization of quantum measurements—a central piece of the puzzle. Specifically, there are at least two points of view from which one can approach measurements: the *informational* and the *physical*. The first one emphasizes that measurements are ways that we humans have to *know things about* the quantum system, to extract information from it in the form of measurement outcomes. The physical point of view, in turn, focuses in the notion that a quantum measurement is a *physical process* just like any other one; even if possibly belonging to a given class with some specific properties. For example, that it must involve at least the system and a macroscopic measuring device, or that the latter must display well defined, distinguishable and meaningful outcomes when the measurement is over.

In this short note, I will present a very simple (almost trivial) *Gedankenexperiment*¹ that seeks to emphasize this latter, physical point of view. My only aim is to show as clearly and as simply as possible that regarding measurements as physical processes is *inescapable*. In particular, this implies that any approach to quantum measurements might involve a combination of informational and physical considerations, but it could never be constituted by the first type *alone*. The reader may feel that everybody must agree—or even that everybody *does* agree—with this thesis, but, given the polyphonic nature of the positions and debates in the field, I think it is better to be safe than sorry.

Also, I will make my point in a very concise way; something which might help to nail down and more fruitfully confront any statement related to the informational vs. physical dichotomy. This extreme simplicity together with the idiosyncratic nuances of my personal approach to the topic are possibly the only original ingredients that the reader will find here. I hope they are enough to make the ride interesting and useful, but if they are not, don't worry. The ride is very short anyway.

2 A simple *Gedankenexperiment*

Let us consider a spin-1/2 quantum system with a Hamiltonian operator which is proportional to the z -component of the spin:

$$\hat{H} = -\alpha\hat{S}_z = -\frac{\alpha\hbar}{2}\hat{\sigma}_z, \quad (2.1)$$

where we have defined the spin- z operator by:

$$\hat{S}_z := \frac{\hbar}{2}\hat{\sigma}_z := \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (2.2)$$

¹ The word is German, and that's why it begins with a capital letter, like all good German nouns. It means “thought experiment”. That is, a very idealized version of an experiment that you only carry out in your imagination, but that you assume to be nevertheless doable in the lab. You also assume that the ideal conditions might never be strictly attained in reality, but approaching them as much as one wants is only a matter of technical ability. Although “Gedanken” is the plural noun meaning “thoughts”, it functions as an adjective here.

Now imagine that we are able to prepare, at time $t_0 = 0$, N copies of the system in the eigenstate $|z+\rangle$ of \hat{S}_z corresponding to the positive eigenvalue $+\hbar/2$. This state is obviously also the ground state of \hat{H} with eigenvalue (energy) equal to $-\alpha\hbar/2$. Therefore, it will evolve in time with just a phase:

$$|\psi(t)\rangle = \exp\left(-i\frac{t}{\hbar}\hat{H}\right)|\psi(0)\rangle = \exp\left(i\frac{\alpha t}{2}\hat{\sigma}_z\right)|z+\rangle = \exp\left(i\frac{\alpha t}{2}\right)|z+\rangle. \quad (2.3)$$

That is, it will remain an energy eigenstate corresponding to the minimum energy $-\alpha\hbar/2$ until we do something else.²

Next (at some time t_1 after preparation), let us measure

$$\hat{S}_x := \frac{\hbar}{2}\hat{\sigma}_x := \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (2.4)$$

on each and every prepared copy. According to the accepted understanding of quantum measurement in textbooks, the result of such an action is that the state $|\psi(t_1)\rangle$ will collapse instantaneously to any one of the two eigenstates $\{|x-\rangle, |x+\rangle\}$ of \hat{S}_x with probabilities $\{P(x-; t_1), P(x+; t_1)\}$ respectively. By Born's rule, we know that:

$$P(x\pm; t_1) = |\langle x\pm | \psi(t_1) \rangle|^2 = \left| \exp\left(i\frac{\alpha t_1}{2}\right) \langle x\pm | z+\rangle \right|^2 = |\langle x\pm | z+\rangle|^2. \quad (2.5)$$

Since

$$|x\pm\rangle = \frac{1}{\sqrt{2}}(|z-\rangle \pm |z+\rangle), \quad (2.6)$$

and of course $\langle z- | z+\rangle = 0$, and $\langle z- | z-\rangle = \langle z+ | z+\rangle = 1$, we have that:

$$P(x\pm; t_1) = \frac{1}{2}. \quad (2.7)$$

That is, half of the times the \hat{S}_x measurement will collapse the state onto $|x-\rangle$, the other half it will collapse it onto $|x+\rangle$, and the probabilities are time independent.

The subsequent evolution of any of these eigenstates of \hat{S}_x can be easily obtained using eq. (2.6) and the fact that the eigenstates $\{|z-\rangle, |z+\rangle\}$ of \hat{S}_z are also eigenstates of \hat{H} with eigenvalues $\{\alpha\hbar/2, -\alpha\hbar/2\}$ respectively:

$$|\psi(t > t_1)\rangle = \frac{1}{\sqrt{2}} \left[\exp\left(-i\frac{\alpha(t-t_1)}{2}\right) |z-\rangle \pm \exp\left(i\frac{\alpha(t-t_1)}{2}\right) |z+\rangle \right]. \quad (2.8)$$

The last step in the *Gedankenexperiment* is to perform an additional measurement, now of \hat{S}_z , at a time $t_2 > t_1$. Orthodox presentations of quantum mechanics indicate again that this will produce the collapse of the state $|\psi(t_2)\rangle$ onto either $|z-\rangle$ or $|z+\rangle$. The corresponding Born probabilities are also time independent, and they can be easily computed:

$$P(z\pm; t_2) = \frac{1}{2}. \quad (2.9)$$

² I am aware that quantum states are not represented by elements of a Hilbert space, but by the corresponding rays, i.e., by the corresponding 1-dimensional linear subspaces spanned by them. To go from rays to vectors (kets), one has to fix the norm and an arbitrary phase. In this paragraph and in everything that follows, all kets are assumed normalized and the phase is chosen to make the expressions as simple as possible.

That is, half of the systems will end up in state $|z-\rangle$ after the \hat{S}_z measurement, and the other half will end up in state $|z+\rangle$.

But wait! States $\{|z-\rangle, |z+\rangle\}$ are also eigenstates of the Hamiltonian operator, which means that any energy measurement at $t_3 > t_2$ is now certain to produce $E = \alpha\hbar/2$ for the first one of them, or alternatively $E = -\alpha\hbar/2$ for the second. This in turn means that, if we produced N copies of our quantum system in eigenstate $|z+\rangle$ at $t_0 = 0$, we began with a situation in which the total energy was

$$E_{\text{tot}}(0) = -N \frac{\alpha\hbar}{2} , \quad (2.10)$$

and we ended up with a situation in which

$$E_{\text{tot}}(t_3) = \frac{N}{2} \frac{\alpha\hbar}{2} - \frac{N}{2} \frac{\alpha\hbar}{2} = 0 . \quad (2.11)$$

That is, we increased the energy of our (collection of) systems by $N\alpha\hbar/2$. Or, if you want to put it more dramatically, *we heated the coffee by just looking at it* (twice).

3 From *Gedanken* to *Laboratorium*

The previous *Gedankenexperiment* is “gedanken” but doable. One possible way of maybe realizing it in a laboratory is through the famous Stern-Gerlach setup [which you can look up in many places, but I have looked up in (Peres, 2002)].

The basic idea is to use some kind of source of neutral particles with spin, generate a collimated beam, and make it go through a region in which there is an inhomogeneous magnetic field; normally produced by a suitable magnet. Combined with some sort of detector after the magnet, this constitutes an experimental embodiment of a quantum “spin measurement”. Indeed, if the mentioned detector is (say) some kind of screen which can be excited anywhere on its 2-dimensional surface, we find that only two discrete spots appear; one corresponding to the eigenstate of the spin operator in the direction of the magnetic field, the other corresponding to the orthogonal one, i.e., to the eigenstate associated to the opposite direction. Since the classical behavior that one would expect (assuming that spin is some kind of magnetic dipole) is to obtain a continuous band between the two spots corresponding to all possible intermediate orientations between completely aligned with the magnetic field and completely anti-aligned to it, the result of this basic Stern-Gerlach setup is typically regarded as demonstrating that spin is *quantized*, i.e., that spin is a quantum property; not a classical one.

This is nice and correct, but I don’t want to prove that quantum mechanics is necessary here. I want to show that, *assuming it is*, we can use it to heat coffee just by looking. So let us use the Stern-Gerlach idea to produce a more complicated—but still sketchy—setup that can move the *Gedankenexperiment* in the previous section closer to a real *Laboratoriumexperiment*.³

A cartoon scheme of the whole thing can be found in fig. 3.1: On the leftmost part, we have some kind of source that emits our spin-1/2 particles in a completely random state; i.e., in the (statistical) state described by a density matrix $\hat{\rho}$ equal to the 2×2 identity matrix \hat{I} . The light-blue arrows in the background represent an homogenous magnetic field in the positive z direction which is assumed to be present at every point of the setup, and which

³ You don’t need to know German to understand this one.

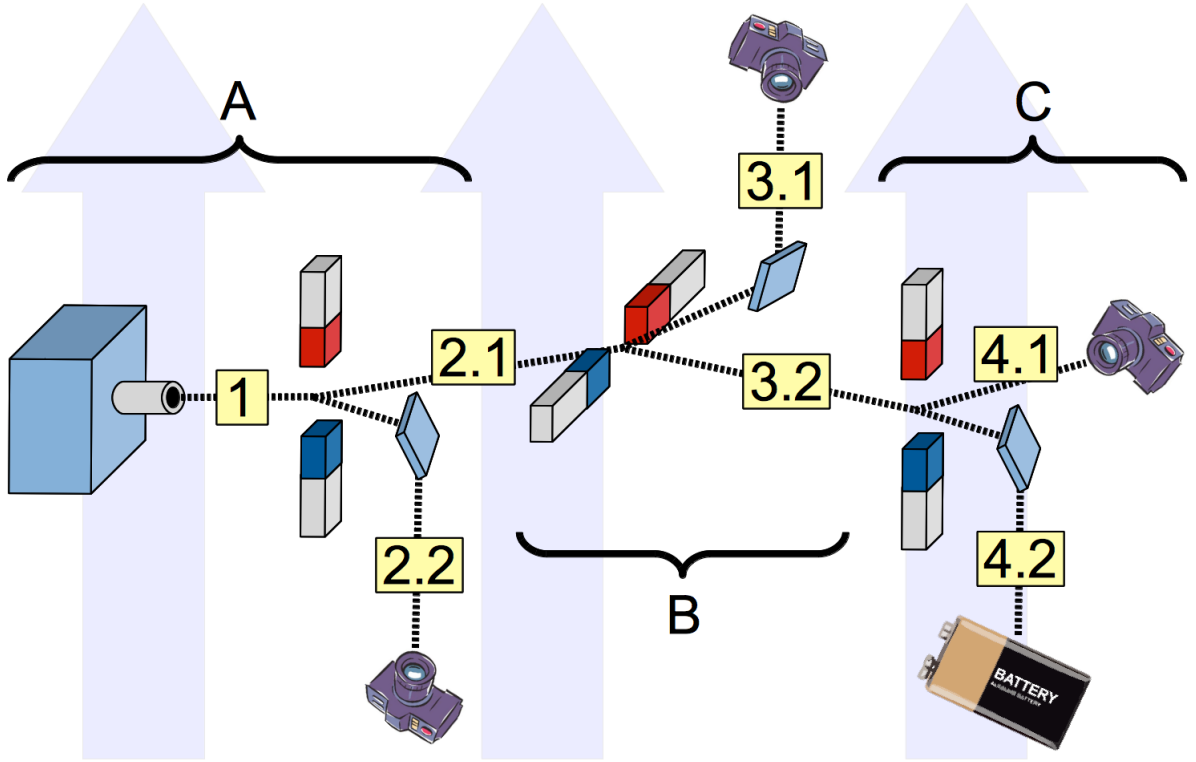


Figure 3.1: Cartoon depiction of the *Laboratoriumexperiment* described in detail in sec. 3 and constituting a possible physical embodiment of the *Gedankenexperiment* in sec. 2.

explains why \hat{H} in eq. (2.1) is proportional to $-\hat{S}_z$. The beam that comes out of the source is labeled as ‘1’ and it is depicted as a discontinuous line; like the rest of beams in the figure.⁴ Beam 1 is assumed to consist of $2N$ particles all of them prepared identically, and we can imagine that the intensity is so low that the particles come out of the source one by one.

Just after the source, we have our first Stern-Gerlach magnet oriented along the z axis. Instead of making the resulting beams impinge on a screen (which would yield the two proverbial spots but would prevent further actions on the particles), we place a mirror at the location where the spot corresponding to $|z-\rangle$ would appear, and a detector (depicted as a camera) just after it. The device formed by the magnet, the mirror and the camera is a physical embodiment of a \hat{S}_z measurement, with the particularity that the systems with spin $z+$ are not “destroyed” but move on. If we add the source to this device, we can regard the whole sub-setup labeled by ‘A’ as a “preparation” of the system onto the state $|z+\rangle$. That is, the state of every particle in beam 2.1 is $|z+\rangle$ and its energy is the minimal one, $-\alpha\hbar/2$ (rigorously speaking, an energy measurement on any particle in beam 2.1 is certain to yield this value). Beam 2.1 can thus be seen as the starting point of the *Gedankenexperiment* in sec. 2, and the total energy it contains is $-N\alpha\hbar/2$, since only half of the original particles come out of the preparing device A. In tab. 1, the number of particles, the states and total

⁴ Of course, quantum mechanics (via the uncertainty principle) banishes well defined trajectories out of existence. As a consequence, all the “beams” I have drawn must be understood as “semiclassical metaphors” which are only intended to help us reason comfortably, and which are in any case *harmless*. Since we materialize the “splitting of beams” at each step by performing a quantum measurement with definite—even if unpredictable—outcomes, all the “beam talk” is in fact consistent and one could dispose of it if extreme purism was intended.

beam	#particles	state	E_{tot}
1	$2N$	$\hat{\rho} = \hat{I}$	0
2.1	N	$ z+\rangle$	$-N\alpha\hbar/2$
2.2	N	$ z-\rangle$	$N\alpha\hbar/2$
3.1	$N/2$	$ x+\rangle$	0
3.2	$N/2$	$ x-\rangle$	0
4.1	$N/4$	$ z+\rangle$	$-N\alpha\hbar/8$
4.2	$N/4$	$ z-\rangle$	$N\alpha\hbar/8$

Table 1: Number of particles, states and total energies of each beam in the experiment discussed in sec. 3 and depicted in fig. 3.1.

energies of each beam are summarized for quick reference.

The next piece of equipment (device B) is constituted first by another magnet, this time oriented along the x axis, which splits beam 2.1 into two new beams: beam 3.1, which is deflected by a mirror and detected by a camera, and beam 3.2, which is allowed to proceed to the last part of the setup. Device B implements a \hat{S}_x measurement, and we assume that we have arranged it in such a way that the state of every particle in beam 3.1 is represented by $|x+\rangle$ and those in beam 3.2 by $|x-\rangle$ (but, in fact, nothing essential changes if we do it the other way around). According to the calculations in sec. 2, the total energy in both beams 3.1 and 3.2 is zero. This already allows us to conclude that, starting from beam 2.1, which had a total energy of $-N\alpha\hbar/2$, we have increased the energy of our collection of particles just by measuring their spin along the x axis. However, in this *Laboratoriumexperiment* I have decided to make this official by performing a final energy measurement.

Device C is just the same as device B, only that it is oriented along the z axis and the particles that come out of the magnet with state $|z-\rangle$ [half of them, as we know through eq. (2.9)] are not allowed to move on, but they are “stored” in some sort of “battery”. Those that come out with state $|z+\rangle$ are simply “detected”.

We can now conclude that, starting from a beam (2.1) with N particles all of them in their ground state, we have managed to produce $N/4$ particles in an excited state. We have stored them in a battery and, provided N is large enough, we may even use them to heat up our coffee. It seems that this setup is producing less energy than the *gedanken* case in sec. 2, but that’s only because we have decided to just “detect” beam 3.1. If we channeled this beam to another device of type C, we would then obtain another $N/4$ particles each one in the excited state of energy $\alpha\hbar/2$, and the real efficiency of our “power station” would attain its theoretical maximum again.

4 Discussion and conclusions

In the previous two sections I have shown how, using the textbook rules for quantum measurements [see, e.g., (Cohen-Tannoudji et al., 1977)], we can transform N spin-1/2 particles in their lowest energy state into $N/2$ particles in the same ground state plus $N/2$ particles in a *higher* energy state. I have shown that it is very easy to do so, and, what is maybe surprising, that we can do it just by performing spin measurements; not by pumping energy

into our system in some obvious way.

That quantum measurements “disturb the system” is a widely known adagio, but I think that explicitly showing that this disturbance can take the form of *a net increase in the system’s energy* is a suggestive way of emphasizing that a quantum measurement might be a very complicated thing, but it must be a very complicated thing of an unavoidable physical nature after all. Otherwise, how could we inject energy onto the system if the only thing we do is measure its properties?

I have chosen energy and not any other magnitude because it seems to me that energy has something “very real” about it, something “very tangible”. After all, we use it to move our cars and planes, and to light and heat our homes. It even has a monetary value! This is all, of course, very informal talk; but not completely so. This line of thought, for example, suggests that—beyond all the quantum conceptual complications—if energy is being injected into the system, we must be providing it *at some point*. Otherwise, I have just invented a machine that can produce dollars out of thin air and this paper is much more important than I first thought.

In the particular setup described in sec. 3, one might guess that device B, i.e., the part of the experiment that implements the measurement of spin in the x direction, must play some role in increasing the energy of the system. Indeed, if we remove it and leave everything else unchanged, we will find that no particle arrives to the “battery”. All N of them will be detected in beam 4.1 as having state $|z+\rangle$ and ground-state energy $-\alpha\hbar/2$; i.e., nothing will have changed with respect to the original beam 2.1. The total energy will also be the same: the minimum one we started with, $-N\alpha\hbar/2$. It thus seems obvious that device B must be injecting at least $\alpha\hbar/4$ joules into the system, and therefore it must be taking this energy from somewhere else. That is, it seems that we must provide at least $\alpha\hbar/4$ joules from the outside to device B if we want that it does its job properly as specified (as if it needs this energy to “rotate” the spins).

One might want to argue that perhaps this energy expenditure and the system’s energy increase are not necessary features of the measurement process, but contingent on the specific way in which the *Laboratoriumexperiment* in sec. 3 has been designed. It is absolutely expected—one might say—that energy is increased; after all, we are not just “measuring spin x ”, we are doing so with a big fat magnet which we need to power somehow. However, the fact that I have established the result in sec. 2 in a completely general way using only the textbook rules of quantum measurements allows us to quickly dismiss this objection, and to confidently bet our money on the truth of the following general constraint:

If you have a spin-1/2 quantum system that has been prepared onto the “spin- z up” eigenstate $|z+\rangle$ of \hat{S}_z and its Hamiltonian is given by $\hat{H} = -\alpha\hat{S}_z$, then it is impossible to measure the spin of the system in the x direction without expending at least $\alpha\hbar/2$ joules.

One can also complain that, in the particular setup in sec. 3, the total energy of the original beam 1 produced by the source is actually zero; hence, no energy is actually being introduced into the system. If we consider our initial system as beam 1, this is indeed so. However, and as it is clear from the discussion of both the *gedanken* and *laboratorium* experiments, this is *not* so if we regard beam 2.1 as our initial system. I cannot find any argument that forbids this choice to be made, so I must temporarily dismiss the objection, even if it looks better than the previous one a priori.⁵

⁵ It helps me to imagine that device A might be located in Alpha Centauri, and we receive beam 2.1

To be forced to expend energy every time we want to perform a (given type of) quantum measurement is of course as physical as anything can get. Since Einstein (and the famous formula in so many T-shirts), we know that having energy is equivalent to existing; at least to *physically* existing. This physical nature of quantum measurements seems absolutely clear from the previous discussion, but what other kind of existence could measurements have had? One might want to answer that they could have existed only as *informational* operations. In fact, that is precisely the way in which they are assumed to exist in classical mechanics, where, as discussed in sec. 1, they can be made subtler and subtler, and no compulsory minimum energy expenditure to carry them out is required by the theory. On the contrary, quantum mechanics, it seems, explicitly includes the—reasonable—requirement that, in order to find out something about a physical system, we have to probe it in the most real of ways, that is, we have to prove it *physically*. And what more obvious signature of physicality that having to expend energy in the process?

This may naturally have some bearing upon a family of approaches to the foundations of quantum mechanics that are sometimes referred to as *informational*. Some say the family was founded by Wheeler and, indeed, his famous motto “it from bit” [i.e., physical existence from information (Wheeler, 1990)] has become almost a mantra in some circles. The basic idea is that most (if not all) of the structure of quantum mechanics is related to information; information about the system for some people, information about the results of possible human interventions (measurements) on it for others. As in any general philosophical viewpoint, the positions of the different researchers come in a wide variety of intensities; from the most radical [claiming, for example, as Wheeler suggested, that the world *is* information], to the more nuanced [see, e.g., (Fuchs, 2001, 2002)].

Informational approaches are very powerful in the practical sense, and they have been indeed instrumental in advancing the most applied quantum fields, such as computation or cryptography. Moreover, I am confident that any solid account of the foundations of quantum mechanics has to take them into account in one way or another. One should not overlook that they seem to deal nicely (at least at first sight) with some of the most troublesome quantum “paradoxes” and the associated conceptual conundrums. However, after noticing that (at least some) measurements require the expenditure of the same kind of physical energy that we use to power our transportation and cities, it looks difficult to argue for the most radical positions that see measurements as something related *only* to the knowledge that some unspecified human has about a given experiment. It seems clear that a measurement must be something that *happens* “out there”, physically, with or without human intervention. And yes, maybe the structure of the theory we use to speak about what happens is related to some kind of information processing and updating. After all, a great deal of what language is useful for is to process and update information, and a physical theory is (mostly) made of language; of precise and sometimes mathematical language, but language in any case. This being said, it looks a little bit of a wild extrapolation from this—almost tautological—reflection to suggest that, just because it seems useful to overlay information theoretic concepts on top of it in order to gain a tighter control of its doings, the world *is* information, or knowledge, or judgements, or informed bets [a position termed *informational immaterialism* in the lucid analysis by Timpson (2010)]. That is, it looks like a wild extrapolation unless the claim that the world is information means *just that*: that our best theory about it has an informational form or flavor (or at least some of its parts have).

after it has traveled 4.367 light years. Only then we increase its energy here on Earth using device B, and store the excited particles in a battery using device C. Of course, as any other imagination crutch, this one is personal, and it is not a complete argument.

In such a case the claim might be true, but only by definition and uninterestingly so.

Finally, let me mention a pair of lines of discussion that have not been tackled here but may be worth pursuing:

On the one hand, notice that I have chosen simplicity over generality in the *Gedankenexperiment* in sec. 2. Clearly, the structure of the whole setup can be abstracted and applied not only to the particular case of a spin-1/2 system with the proposed sequence of measurements, but to the more general situation of, say, any quantum system that we prepare at $t_0 = 0$ in the ground state of the corresponding Hamiltonian, and on which we perform a measurement of any operator which does not commute with the said Hamiltonian. This will inevitably result in the collapse onto a state with non-zero projection in the subspace orthogonal to the ground-state ray, and thus the total energy of a large collection of these systems will be increased by the measurement. It may be interesting to analyze the workings and properties of this more general case, and possibly also of further generalizations (e.g., other preparations at $t_0 = 0$).

On the other hand, it is important to point out that I have circumscribed in this note to the textbook notion of projective, von Neumann measurements exclusively. If this constraint is lifted and more general ways of obtaining information about our quantum system are allowed, it is very likely that substantial modifications to the whole analysis will have to be made. For example, one can consider generalized measurements based on positive-operator valued measures (POVMs) [see, e.g., (Barnett, 2009, Nielsen and Chuang, 2010)], or the so-called weak measurements (Aharonov et al., 1988, Vaidman, 2009), or even non-formalized operations such as asking the person that prepared the state how she did it. I have shown that (at least some) textbook quantum measurements require a finite amount of energy to occur, and thus must be regarded as physical processes. Whether or not the same can be said about other varieties of measurement is a question for the future.

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References

- Aharonov, Y., Albert, D. Z., and Vaidman, L. (1988). How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100. *Phys. Rev. Lett.*, 60:1351–1354.
- Barnett, S. M. (2009). *Quantum information*. Oxford University Press.
- Cohen-Tannoudji, C., Diu, B., and Laloë, F. (1977). *Quantum mechanics. Volume 1*. Wiley.
- Echenique-Robba, P. (2013). Shut up and let me think! Or why you should work on the foundations of quantum mechanics as much as you please. [<http://arxiv.org/abs/1308.5619>].
- Fuchs, C. A. (2001). Quantum foundations in the light of quantum information. In Gonis, T., Gonis, A., and Turchi, P. E. A., editors, *Decoherence and its implications in quantum computation and information transfer*, page 45. IOP Press. [<http://arxiv.org/abs/quant-ph/0106166>].

- Fuchs, C. A. (2002). Quantum mechanics as quantum information (and only a little more). [<http://arxiv.org/abs/quant-ph/0205039>].
- Nielsen, M. A. and Chuang, I. L. (2010). *Quantum computation and quantum information*. Cambridge University Press.
- Peres, A. (2002). *Quantum theory: Concepts and methods*. Kluwer Academic Publishers.
- Timpson, C. G. (2010). Information, immaterialism, instrumentalism: Old and new in quantum information. In Bokulich, A. and Jaeger, G., editors, *Philosophy of quantum information and entanglement*, pages 208–228. Cambridge University Press. [<http://bit.ly/1gxp7Rk>].
- Vaidman, L. (2009). Weak value and weak measurements. In Greenberger, D., Hentschel, K., and Weinert, F., editors, *Compendium of quantum physics. Concepts, experiments, history and philosophy*, pages 840–842. Springer. [<http://arxiv.org/abs/0706.1348>].
- Wheeler, J. A. (1990). Information, physics, quantum: The search for links. In Zurek, W. H., editor, *Complexity, entropy, and the physics of information*, pages 3–28. Addison-Wesley.